Solar forced Dansgaard-Oeschger events and their phase relation with solar proxies

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North Atlantic climate during glacial times was characterized by large-amplitude switchings, the Dansgaard-Oeschger (DO) events, with an apparent tendency to recur preferably in multiples of about 1470 years. Recent work interpreted these intervals as resulting from a subharmonic response of a highly nonlinear system to quasi-periodic solar forcing plus noise. This hypothesis was challenged as inconsistent with the observed variability in the phase relation between proxies of solar activity and Greenland climate. Here we reject the claim of inconsistency by showing that this phase variability is a robust, generic feature of the nonlinear dynamics of DO events, as described by a model. This variability is expected from the fact that the events are threshold crossing events, resulting from a cooperative process between the periodic forcing and the noise. This process produces a fluctuating phase relation with the periodic forcing, consistent with proxies of solar activity and Greenland climate.

1. Introduction

Climate archives from the North Atlantic region show repeated shifts in glacial climate, the Dansgaard-Oeschger (DO) events [Grootes et al., 1993; Andersen et al., 2006]. During Marine Isotope Stages 2 and 3 the intervals between the events exhibit a tendency to coincide approximately with multiples of 1470 years [Rahmstorf, 2003], as depicted in figure 1. The statistical significance of this pattern and the responsible mechanism, however, is still a matter of debate [Ditlevsen et al., 2005]. Several hypotheses were proposed to explain the timing of DO events. One of these relates the events to two century-scale solar cycles with periods close to 1470/7 (=210) and 1470/17 (≈87) years [Braun et al., 2005], the so-called De Vries/Suess [Wagner er al., 2001] and Gleissberg [Peristykh and Damon, 2003] cycles. Support for a leading solar role comes from deep-sea sediments, which indicate that during the Holocene century-scale solar variability was a main driver of multi-centennial scale climate changes in the North Atlantic region [Bond et al., 2001].

Recently the phase relation between solar variability (deduced from ^{10}Be) and 14 DO events was analyzed [Muscheler and Beer, 2006]. A relation far from fixed was found and was interpreted as being in contradiction to Braun et al.'s hypothesis [Braun et al., 2005]. While in linear systems a constant phase relation between the forcing and the response is expected, such a relation does not necessarily exist in non-linear systems. But climate records and ocean-atmosphere models [Ganopolski and Rahmstorf, 2001], which are not yet suitable for statistical analyses on DO events because of their large computational cost, suggest that the events represent switches between two climate states, consequently implying an intrinsically non-linear dynamical scenario. Thus, to interpret the reported lack of a fixed phase relation between the DO events and solar proxies, it is crucial to analyze their phase relation in simple models.

2. A Simple Model of DO Events

Here we investigate this phase relationship in a very simple model of DO events. A comprehensive description of this model has already been published before, including a detailed discussion of its geophysical motivation and its applicability, as well as a comparison with a much more detailed ocean-atmosphere model [Braun et al., 2007]. In the simple model, which was derived from the dynamics of the events in that ocean-atmosphere model, DO events represent repeated switches between two possible states of operation of a bistable, excitable system with a threshold (Figure 2). These states correspond to cold and warm periods of the North Atlantic region during DO cycles. The switches are assumed to occur each time the forcing function (f), which mimics the solar role in driving DO events, crosses the threshold (T). Transitions between the two states are accompanied by an overshooting of the threshold, after which the system relaxes back to its respective equilibrium following a millennial-scale relaxation.

The rules for the transitions between both states are illustrated in figure 2. It is assumed that the threshold function T is positive in the interstadial ("warm") state and negative in the stadial ("cold") state. A switch from the stadial state to the interstadial one is triggered when the forcing f is smaller than the threshold function, i.e. when $f(t) < T(t)$. The opposite switch occurs when $f(t) > T(t)$. During the switches a discontinuity in the threshold function is assumed, i.e. T overshoots and takes a non-equilibrium value (A_0 during the shift into the stadial state, A_1 during the opposite shift). Afterwards, T approaches its new equilibrium value (B_0 in the stadial state, B_1 in the interstadial state) following a millennial scale relaxation process:

$$\frac{dT}{dt} = -\frac{T - B_s}{\tau_s}. \quad (1)$$

Here, τ_0 and τ_1 represent the relaxation time in the stadial state (s=0) and in the interstadial state (s=1), respectively.

Both the overshooting relaxation assumption and the transition rules in our simple model are a first order approximation of the dynamics of DO events in a coupled ocean-atmosphere model [Ganopolski and Rahmstorf, 2001]. In that model the events also represent threshold-like switches in a system with two possible states of operation (corresponding to two fundamentally different modes of deep water formation in the North Atlantic) and with an overshooting in the stability of the system during these shifts [Ganopolski and Rahmstorf, 2001; Braun et al., 2007]. Analogous to the simple model, switches from the stadial mode into the interstadial one are triggered by sufficiently large negative forcing anomalies (i.e. by a reduction in the surface freshwater flux to the North Atlantic that exceeds a certain threshold value), whereas the opposite shifts are triggered by sufficiently large positive forcing anomalies (i.e. by an increase in the freshwater flux that exceeds a certain threshold value). It has further been demonstrated that the simple model is able to reproduce the timing of DO events as simulated with the ocean-atmosphere model, as well as the occurrence of non-linear resonance phenomena such as stochastic resonance and ghost resonance, which were shown to be properties exhibited by that model [Ganopolski and Rahmstorf, 2002; Braun et al., 2005; Braun et al., 2007].

An obvious advantage of the conceptual model compared with the ocean-atmosphere model is its low computational cost, which allows for extensive statistical analyses on the timing of DO events. All model-parameter values chosen here are the same as in two earlier publications ($A_0 = -27$ mSv, $A_1 = 27$ mSv, $B_0 = -9.7$ mSv, $B_1 = 11.2$ mSv, $\tau_0 = 1200$ years, $\tau_1 = 800$ years; 1 mSv = 1 milli-Sverdrup = 10^3 m^3/s) [Braun et al., 2005; Braun et al., 2007].

3. Testing Fixed Phase Relationships

To test the assumption of a fixed phase relationship between solar-forced DO events and solar variability, we here drive our model by a simple input consisting of noise $\sigma \cdot n(t)$ and of two sinusoidal cycles with equal amplitudes A:

$$f(t) = -A \cdot (\cos[\frac{2\pi t}{T_1}] + \cos[\frac{2\pi t}{T_2}]) + \sigma \cdot n(t). \tag{2}$$

σ is the standard deviation of the noise and $n(t)$ the standard unit variance white noise, with a cutoff frequency of 1/50 years (figure 3). Following Braun et al. [2007] the cutoff is used to account for the fact that the model shows an unrealistically large sensitivity to decadal-scale or faster forcing. In analogy to Braun et al. [2005] the periods of the two cycles are chosen to be $T_1 = 1470/7$ (=210) years and $T_2 = 1470/17$ ($\approx$86.5) years, i.e. close to the leading spectral components of the solar De Vries and Gleissberg cycles.

In the simulations shown in Figure 4 we use three different signal-to-noise ratios (SNR): A = 8 mSv and $\sigma = 5.5$ mSv ($SNR \approx 2.1$), A = 5 mSv and $\sigma = 8$ mSv ($SNR \approx 0.4$), A = 3 mSv and $\sigma = 9$ mSv ($SNR \approx 0.1$). For all of these, the waiting time distribution of the simulated events is centered around a value of 1470 years, with several peaks of only decadal-scale width (figure 4). The relative position of these peaks is well understood in the context of the ghost resonance theory [Chialvo et al., 2002; Chialvo, 2003; Calvo and Chialvo, 2006; Braun et al., 2007]. The peaks result from constructive interference between the two sinusoidal forcing cycles which produces particularly large magnitude variations in the bi-sinusoidal forcing and – when noise is added – leads to favored transitions at the corresponding waiting times. Depending on the relative amplitude values of the noise and the periodic forcing this synchronization is more or less efficient, as is seen for the different signal-to-noise ratios in Figure 4. Even for the lowest ratio, however, the synchronization is still notable. The waiting time distributions shown in figure 4 are thus almost symmetrically centered around a preferred value of 1470 years because the sinusoidal cycles enter in phase every 1470 years, creating forcing peaks of particularly large magnitude. This 1470-year repeated coincidence of the bi-sinusoidal forcing, however, does not show up as a corresponding forcing frequency, since no sinusoidal cycle with that period is present. Thus, when linear spectral analysis is performed on the forcing, only the two century-scale sinusoidal cycles are detected as outstanding components.

Despite the robustness of the synchronization effect, none of the two sinusoidal cycles in our forcing shows a fixed phase relationship with *all* of the simulated DO events, due to the presence of noise and the existence of a threshold. In our model, a fixed phase relation can only be present in the low noise limit (i.e. either for $\sigma \to 0$ [with a supra-threshold bi-sinusoidal forcing] or for the lowest noise level that still enables repeated threshold crossings [with a sub-threshold bi-sinusoidal forcing], thus corresponding to DO events with extremely long waiting times), compare third column in figure 4. Even for the largest of the three signal-to-noise ratios in our simulations, the events thus only show a tendency to cluster around a preferred phase of the two sinusoidal cycles. Outliers, however, can still occur in almost opposite phase, at least once over a sufficiently large number of events in the simulation. For the highest signal-to-noise ratio, for example, there is still a probability of about 35 percent to find at least one out of 14 events in opposite phase (i.e. outside of the interval $[-\pi/2, +\pi/2]$). And for the other two signal-to-noise ratios, the corresponding probability is even much higher (i.e. 92 percent and 99.5 percent, respectively). Since a fixed phase relationship between the simulated events and the forcing cycles does not even exist in our very simple model system, it appears unrealistic to us to assume the existence of such a relationship in the climate system. Thus, the reported lack of a fixed phase relationship between DO events and solar variability (deduced from ^{10}Be) would also be expected with the proposed ghost resonance solar forcing. We note that superimposed epoch analyses of 14 simulated events can produce forcing-response relations similar to the one reported by Muscheler and Beer [2006]: The onset of the superimposed events (at $t = 0$ in the fourth column in figure 4) typically coincides with a minimum in the averaged bi-sinusoidal forcing which, however, is not more pronounced than other minima and is highly damped as compared with the unaveraged forcing. A considerable statistical spread exists in the magnitude of this damping because the small number (14) of events is not yet sufficient to infer reliable information concerning the average phase fluctuation between the input and the output.

This lack of phase correlation between forcing and response is explained by the threshold character of DO events: The simulated events are triggered when the total forcing (the sum of the two sinusoidal cycles and the noise) crosses the threshold function. Some of the threshold-crossings are in the first place caused by constructive interference of both cycles. These events coincide with near-minima of the two forcing cycles. Other threshold-crossings are, however, in the first place caused by constructive interference of just one cycle and noise. These events thus coincide with a near-minimum of only that cycle (compare figure 3), whereas a fixed phase-relation with the second cycle does not necessarily exist. And at least for low signal-to-noise ratios, some of the threshold crossings are in the first place caused by the noise alone. These events thus do not show a fixed phase relation with any of the two forcing cycles.

The inherently nonlinear noisy synchronization mechanism exhibited by our model is not unique to DO events. In fact, it has originally been proposed to explain the perception of the pitch of complex sounds [Chialvo et al., 2002; Chialvo, 2003] and, as a general concept, has already been used to describe theoretically and experimentally similar dynamics in other excitable [Calvo and Chialvo, 2006] or multi-stable systems with thresholds, e.g. in lasers [Chialvo et al., 2002; Chialvo, 2003; Buldu et al., 2003]. Because of the fact that the leading output frequency is absent in the input, this type of resonance is called ghost stochastic resonance.

4. Conclusions

We here used a simple model of DO events, driven by a bi-sinusoidal forcing plus noise, to show that a fixed phase relation between the forcing cycles and *all* simulated events does not exist, apart from the unrealistic low noise limit. As argued above, in this model the fluctuations in the phases between the forcing and the response are related to the process giving rise to the transition itself. Each event is generated by a threshold crossing resulting from a cooperative process between the two periodic driving forces (i.e., the centennial-scale input cycles) and the stochastic fluctuations. In this nonlinear scenario, as we showed explicitly in our simulations, millennial-scale events with fixed input-output phase relations are impossible for any nonzero noise amplitude.

While one could disagree on the interpretation and the statistical significance of the pattern described in Figure 1, our results show that the reported lack of a fixed phase relationship between 14 DO events and solar proxies is consistent with the suggested solar role in synchronizing DO events. At the same time our results have further implications for a second so-far unexplained oscillation during Pleistocene climate, i.e. the glacial-interglacial cycles, which also show strong indications for the existence of threshold-like dynamics during glacial terminations [Paillard, 1998; Huybers and Wunsch, 2005]: Since the existence of a causal relation between threshold-crossing events and their quasi-periodic forcing does not necessarily imply the existence of a clear phase relation over *all* events, the lack of such a relation between glacial terminations on one hand and the orbital eccentricity and precession cycles on the other hand is not sufficient to reject a leading role of these cycles during terminations, in contrast to the interpretation proposed by Huybers and Wunsch [2005]. More insight in the cause of Pleistocene climate cycles might thus be gained from more adequate statistical approaches, based e.g. on Monte-Carlo simulations [Ditlevsen et al., 2007] with simple models [Paillard, 1998; Braun et al., 2007] that mimic the nonlinear dynamics which seems to be relevant during these oscillations.

Acknowledgments. H.B. thanks A. Ganopolski for instructive discussions on the dynamics of DO events and A. Svensson for his hospitality at the Niels Bohr Institute. H. B. was funded by Deutsche Forschungsgemeinschaft (project MA 821/33).

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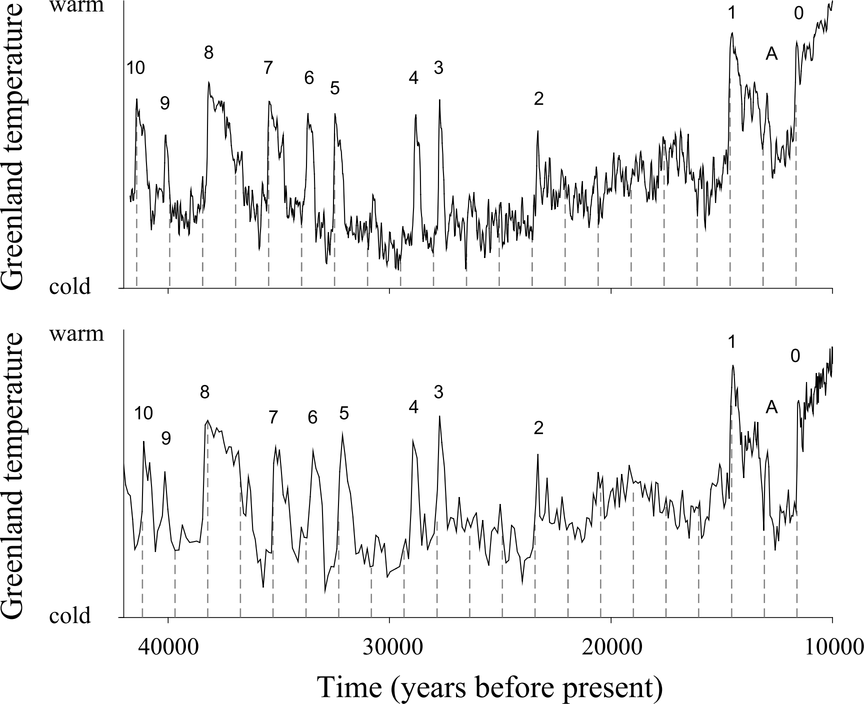
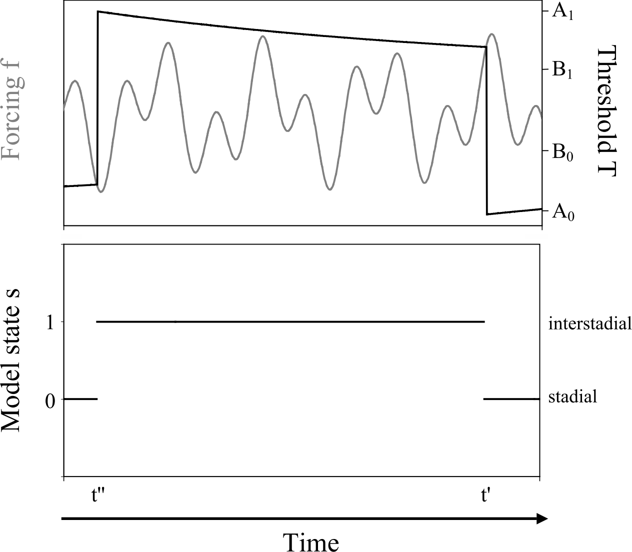
Figure 1. DO events as seen in two ice cores from Greenland. Top: NGRIP. Bottom: GISP2. Labels mark the events 1-10, the Allerød (A) and the end of the Younger Dryas (0). Dashed lines are spaced by 1470 years.

Figure 2. Dynamics of DO events in the model. Top: Forcing f (grey) and threshold function T (black). Bottom: Model state s (s=0: "cold" state, $s = 1$: "warm" state). A switch from the cold to the warm state is triggered when $f < T$, which happens in this example at time t''. During the transition, interpreted as the start of a DO event, T overshoots and relaxes back towards its new equilibrium B_1 ($B_1 > 0$) following a millennial time scale. The events are terminated by a switch back to the cold state, which is triggered when $f > T$ (at time t' in the figure). Again, T overshoots and approaches its new equilibrium B_0 ($B_0 < 0$).

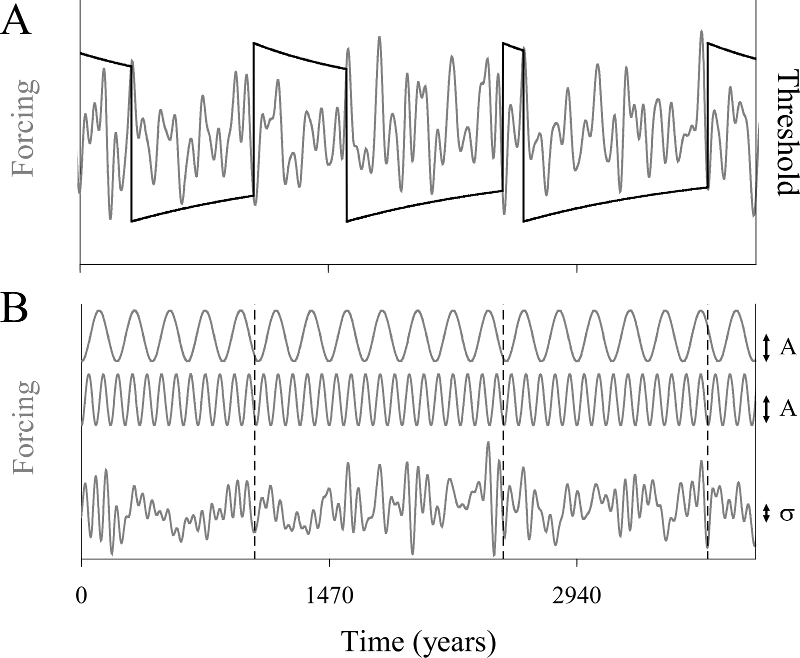
Figure 3. Forcing and response. The forcing consists of two added sinusoidal cycles with equal amplitudes (A = 8 mSv) and white noise (σ = 5.5 mSv). (A) Total forcing f (grey) and threshold function T (black). (B) Forcing components (grey); from top to bottom: 210-year cycle, 86.5-year cycle, noise. Dashed lines indicate the onset of the simulated DO events. Despite the tendency of the three events to recur approximately every 1470 years, only the first two events coincide with minima of the 210-year cycle. The third event, in contrast, occurs closely after a maximum of that cycle.

Figure 4. Waiting time distribution of the simulated events and their phase relation with the forcing. The amplitude of the two sinusoidal forcing cycles and the standard deviation of the noise are: A = 8 mSv, σ = 5.5 mSv (top); A = 5 mSv, σ = 8 mSv (middle); A = 3 mSv, σ = 9 mSv (bottom). First column: normalized distribution of the spacing between successive DO events in the simulation. Second column: probability distribution of the phase relation between the onset of the events and the 210-year cycle (zero corresponds to a start of the events at the minimum of that cycle, $\pm\pi$ to a start at the maximum). Third column: Rayleigh's R value [Ditlevsen et al., 2007], a measure for the phase correlation between the forcing cycles and the simulated events, as a function of the noise level σ ($R = \frac{1}{N}|\sum_{n=1}^{N} \cos\frac{2\pi t_n}{T_1} + i \cdot \sin\frac{2\pi t_n}{T_1}|$, where t_n denotes the timing of the events, $T_1 = 210$ years and $N \rightarrow \infty$). R is 1 if and only if a fixed phase relation exists between the 210-year forcing cycle and the events. Fourth column: Superposition of the model response (i.e. of the state variable s) and of the bi-sinusoidal forcing over a series of 14 simulated events, aligned by the onset of the events following Muscheler and Beer [2006]. The unaveraged bi-sinusoidal forcing is normalized with maximum and minimum values of ± 1.

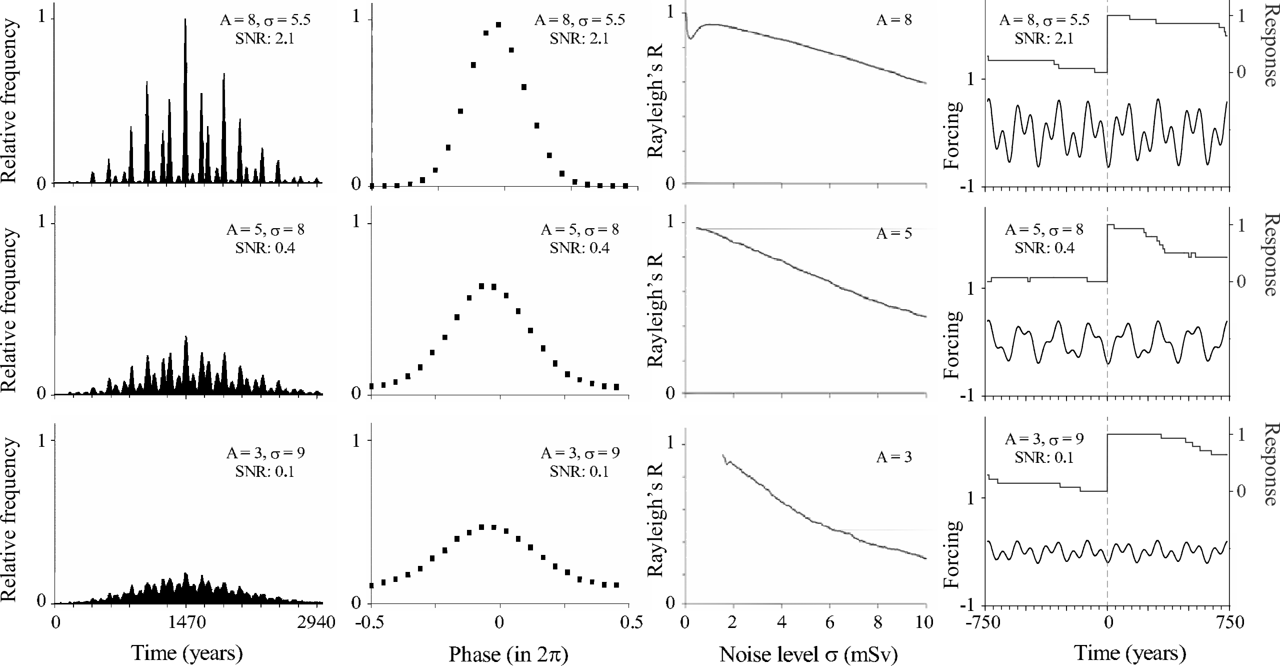

warm
cold
Greenland temperature
10
9
8
7
6
5
4
3
2
1
A
0
40000
30000
20000
10000
Time (years before present)



Forcing f
Threshold T
A1
B1
B0
A0
Model state s
1
0
interstadial
stadial
t''
t'
Time



A
Forcing
Threshold
B
Forcing
A
A
σ
0
1470
2940
Time (years)



Relative frequency
A = 8, σ = 5.5
SNR: 2.1
A = 5, σ = 8
SNR: 0.4
A = 3, σ = 9
SNR: 0.1
Time (years)
Phase (in 2π)
Rayleigh's R
A = 8
A = 5
A = 3
Noise level σ (mSv)
Forcing
Response
Time (years)